Problem Set – Internet Economics course 2009/10

Submission date: March 1st (in class)

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Instructions:

- Please print the problem set and answer the questions in the allotted space. Please do not add extra pages.
- Please answer all questions. All questions have equal weight.
- When you are asked to write a number, please only write a number with no supporting text.
- When you are asked to write text, be brief. Write in Hebrew.
- Q. 1 is about single-item auctions, discusses in classes 3 and 4.
- Q. 2 is about VCG mechanisms, discussed in classes 5 and 6.
- Q. 3 is about diffusion in social networks, discusses in class 11.
  See also chapter 19 of the book "Networks, Crowds, and Markets" by Easley and Kleinberg,
  in http://www.cs.cornell.edu/home/kleinber/networks-book.
- Q. 4 is about sponsored search auctions, discusses in classes 7 and 8.
  See also chapter 15 of the book "Networks, Crowds, and Markets" by Easley and Kleinberg,
  in http://www.cs.cornell.edu/home/kleinber/networks-book
  or Chapter 28 of the book Algorithmic Game Theory.

Good luck!

1 Question 1: Third-price auctions

A seller is selling a single good to a set of n bidders ($n > 3$). Consider the following auction ("third-price auction"):

- Each bidder submits a bid for the item.
- The bidder who submitted the highest bid wins the item (in case of a tie, one bidder with a highest bid is chosen arbitrarily).
- The winner pays the *third*-highest bid.

Prove or disprove: *the third-price auction is truthful in dominant strategies.*

Remark: Disproving the claim requires showing a scenario where misreporting the true value can benefit some bidder. Proving the claim requires a set of arguments (like the arguments given in class for the second-price auction) showing that a bidder will never benefit from lying.

2 Question 2: Weaknesses of VCG mechanisms

A seller is selling two items, a and b. Two bidders are interested in buying the items:

- Bidder 1 is willing to buy the two items together for \$2, but is not interested at all in buying any item separately.
  That is, $v_1(ab) = 2, v_1(a) = 0, v_1(b) = 0$.
- Bidder 2 is willing to buy each item, or the two items together, for \$2. That is, $v_2(ab) = 2, v_2(a) = 2, v_2(b) = 2$.

1. Describe an efficient allocation of the two items?

   Answer: Bidder 1 receives items ____________ and Bidder 2 ____________

2. In a VCG auction, how will the items will be allocated? How much each one of the bidders will pay in a VCG auction?

   Answer: In the VCG auction,
   Allocation: Bidder 1 receives items ____________ and Bidder 2 ____________

   Payments: Bidder 1 pays ____________ and Bidder 2 pays ____________

3. A third bidder enters the market. The third bidders has preferences identical to bidder 2. That is, $v_3(ab) = 2, v_3(a) = 2, v_3(b) = 2$. How will the items be allocated now in a VCG auction, and what are the payments for the bidders?

   Answer:
   Allocation: Bidder 1 receives items ____________, Bidder 2 ____________ and Bidder 3 ____________.

   Payments: Bidder 1 pays ____________, Bidder 2 pays ____________ and Bidder 3 pays ____________

4. An auction is *revenue-monotone* if expanding the market (that is, adding bidders to the market) can only increase the seller's revenue. Given your answer to the previous item, is the VCG auction monotone?

Answer: Yes / no (Circle the right answer)

5. Shill bidding (or "false-name" bidding) is when a bidder submits additional bids as a fake bidder.

   Consider now the original two bidders 1 and 2. Given your answers to the previous items, show that Bidder 2 can benefit from shill bidding.

   Answer: Bidder 2 can benefit from shill bidding by ____________

   ____

   ____

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3 Question 3: Diffusion in Social Networks

Suppose that initially everyone is using behavior B in the social network in Figure 1, and then a new behavior A is introduced. This behavior has a threshold of q = 1/2: any node will switch to A if at least 1/2 of its neighbors are using it.

- Find a set of three nodes in the network with the property that if they act as the three initial adopters of A, then it will spread to all nodes. (In other words, three nodes who are capable of causing a cascade of adoptions of A.)

  Answer: The three nodes are ____________________________________

- Find three clusters in the network, each of density greater than 1/2, with the property that no node belongs to more than one of these clusters.

  Answer: The clusters are ____________________________________

- How does your answer to (c) help explain why there is no set consisting of only two nodes in the network that would be capable of causing a cascade of adoptions of A? (I.e., only two nodes that could cause the entire network to adopt A.)

  Answer: ____

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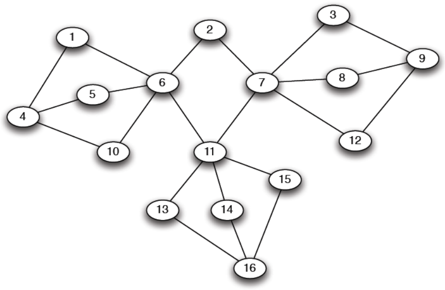

Figure 1: This is the network discussed in Question 3.

4 Question 4

Suppose a search engine has two ad slots that it can sell, and seller is using the Generalized Second-Price auction (GSP). Slot 1 has a clickthrough rate of 0.12 and slot 2 has a clickthrough rate of 0.05. There are three advertisers who are interested in these slots. Advertiser x values clicks at 15 per click, advertiser y values clicks at 13 per click and advertiser z values clicks at 9 per click.

Compute a set of bids for the advertisers that are an equilibrium in the normal-form game created by the auction (as presented in class).

Answer: In equilibrium,

Advertiser x bids ____________________,

Advertiser y bids ____________________

and Advertiser z bids ____________________.